Semidefinite Relaxation-based Beamforming Algorithm for IRS-assisted Wireless Network

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Intelligent reflective surface (IRS) is an innovative and innovative technology for cost-effective achievement of future frequency and energy-efficient wireless communication. In particular, IRS consists of a number of low-cost manual elements that can independently reflect incident signals with adjustable phase shifts to achieve cooperative passive beamforming without requiring transmission radio frequency (RF) chains. In this project, we formulate and solve a novel problem to minimize the uplink total transmission power of users by jointly optimizing their transmission beamforming and reflective beamforming in IRS by control the Phase Shift under the users' individual signal-to-interference-plus-noise ratio (SINR) constraints. In this paper we propose an distributed algorithm in an alternation manner based on semidefinite relaxation (SDR) to solve the problem that having a non-convex constraints. Our proposed algorithm can obtain both a performance upper bound and a high-quality approximation solution.

I. Introduction

Recently, the need to use Intelligent Reflecting Surface (IRS) for the development of wireless communication has drawn attention. In particular, IRS is a planar arrangement consisting of a number of passive elements, where each element can induce a specific phase shift independently of incident electromagnetic waves. By smartly adjusting the phase shift of all passive elements in the IRS, reflected signals can be added consistently with signals coming from different paths of the desired receiver to increase received signal power, or destructively from unintended receivers to enhance security/privacy.

In our research, we consider an IRS-enabled multi-user single-input multiple-output (SIMO) uplink communication system in a single cell, as shown in Figure 1. Here, multi-antenna AP serves multiple single antenna users with the help of the IRS. Typically, APs receive uplink signals from both user-AP (direct) and user-IRS-AP (reflective) links, so we simultaneously optimize each user's (active) transmission beamforming and (manual) reflection beamforming by IRS phase shifter to minimize the total transmission power of the entire user, under a given set of signal-to-interference-plus-noise ratio (SINR) constraints.

However, our design problem is typically difficult to solve optimally due to the non-convex SINR constraints as well as the signal unit-modulus constraints imposed by passive phase shifters. To address the non-convexity of the considered problem, we propose an distributed algorithm in an alternation manner based on semidefinite relaxation (SDR) to obtain both a performance upper bound and a high-quality approximation solution. The key idea is that AP and IRS independently coordinate transmission beamforming and phase shift until convergence is reached.

II. Problem Formulation

Let $W = [w_1, \cdots, w_K] \in \mathbb{C}^{M\times K}$, $H_r = [h_{r,1}, \cdots, h_{r,K}] \in \mathbb{C}^{N\times K}$, and $H_d = [h_{d,1}, \cdots, h_{d,K}] \in \mathbb{C}^{M\times K}$ In this paper, we aim to minimize the total transmission power of all users under the individual SINR constraints of each user by simultaneously optimizing the transmission beamforming of all users and the reflective beamforming in IRS. Therefore, this problem is formulated as follows

$$\mathcal{P}1: \quad \min_{P,W,\theta} \sum_{k=1}^{K} P_k \tag{1a}$$

$$\text{s.t.} \quad \frac{|w_k^H(h_{b,k} + G_b\bar{h}_k\theta)|^2 P_k}{\sum_{j\neq k}|w_k^H(h_{b,j} + G_b\bar{h}_j\theta)|^2 P_j + \sigma_k^2} \geq \gamma_k \tag{1b}$$

$$||w_k||^2 = 1 \tag{1c}$$

$$|\theta_{l,n}| = 1 \tag{1d}$$

$$0 \leq P_k \leq P_k^{max} \tag{1e}$$

where γ_k is the minimum SINR QoS of user k. Problem P1 is hard to solve due to the non-convex constraint in (2) where the transmit beamforming and phase shifts are coupled. In the following sections, we apply SDR and alternating optimization techniques, respectively, to approximately solve P1 for a single-user case, which are then generalized to the multi-user case.

III. SINGLE-USER SYSTEM

A. SDR

By using MRC, the optimal receive beamforming, we can easily obtain $w^* = \frac{(h_b+G_b\bar{h}\theta)^H}{||h_b+G_b\bar{h}\theta||}$ In that situation, redefining problem P1 is

$$\mathcal{P}2: \quad \min_{P,\theta} \; P \tag{2a}$$

$$\text{s.t.} \quad P||h_b+G_b\bar{h}\theta)||^2 \geq \gamma\sigma^2 \tag{2b}$$

$$|\theta_{l,n}| = 1 \tag{2c}$$

$$0 \leq P \leq P^{max} \tag{2d}$$

It is not difficult to verify that the optimal transmit power satisfies $P^* = \frac{\gamma\sigma^2}{||h_b+G_b\bar{h}\theta)||^2}$. And minimize p^* is equivalent to maximize $||h_b+G_b\bar{h}\theta)||^2$

$$\mathcal{P}3: \quad \max_{\theta} \; ||h_b+G_b\bar{h}\theta)||^2 \tag{3a}$$

$$\text{s.t.} \quad |\theta_{l,n}| = 1, \qquad \theta = [\theta_{1,1},\cdots,\theta_{1,N},\cdots,\theta_{L,1},\cdots,\theta_{L,N}] \tag{3b}$$

Then let $\Gamma = G_b h \in \mathbb{C}^{M\times LN} \Rightarrow ||h_b+\Gamma\theta||^2$

$$\mathcal{P}4: \quad \max_{\theta} \; \theta^H\Gamma^H\Gamma\theta + \theta^H\Gamma h_b + (\Gamma^H h_b)^H\theta + ||h_b||^2 \tag{4a}$$

$$\text{s.t.} \quad |\theta_{l,n}|^2 = 1 \tag{4b}$$

By introducing an auxiliary variable t, we have

$$\mathcal{P}5: \quad \max_{\theta} \; \theta^H R\theta + ||h_b||^2 \tag{5a}$$

$$\text{s.t.} \quad |\theta_{l,n}|^2 = 1 \tag{5b}$$

where

$$R = \begin{bmatrix} \Gamma^H\Gamma & \Gamma^H h_b \\ h_b H\Gamma & 0 \end{bmatrix}, \quad \bar{\theta} = \begin{bmatrix} \theta \\ t \end{bmatrix}$$

Note that $\bar{\theta}^H R\bar{\theta} = Tr(R\bar{\theta}\bar{\theta}^H)$. Define $\Phi = \bar{\theta}\bar{\theta}^H$ which satisfies $\Phi \succeq 0, rank(\Phi) = 1$

$$\mathcal{P}6: \quad \max_{\Phi} \; Tr(R\Phi) \tag{6a}$$

$$\text{s.t.} \quad \Phi \succeq 0, \tag{6b}$$

$$\Phi_{t,t} = 1, \qquad t = 1, 2, \cdots, LN+1 \tag{6c}$$

B. Alternating optimization

① Give W

$$\mathcal{P}7: \quad \min_{P,\theta} \; P \tag{7a}$$

$$\text{s.t.} \quad |W^H(h_b + G_b\bar{h}\theta)|^2 P \geq \gamma\sigma^2 \tag{7b}$$

$$|\theta_{l,n}| = 1 \tag{7c}$$

$$0 \leq P \leq P^{max} \tag{7d}$$

Then we can easily achieve $P^* = \frac{\gamma\sigma^2}{||h_b + G_b\bar{h}\theta)||^2}$. And using this, the above problem can be transformed as below.

$$\mathcal{P}8: \quad \max_{\theta} \; |W^H(h_b + G_b\bar{h}\theta)|^2 \tag{8a}$$

$$\text{s.t.} \quad |\theta_{l,n}| = 1 \tag{8b}$$

$|W^H(h_b+G_b\bar{h}\theta)| = |W^H h_b + W^H G_b\bar{h}\theta| \leq |W^H h_b| + |W^H G_b\bar{h}\theta|$ this equality holds if $arg(W^H h_b) = arg(W^H G_b\bar{h}\theta) = \varphi_0$

Let $a^H = W^H G_b\bar{h}$, then $W^H G_b\bar{h}\theta = a^H\theta$

$$\mathcal{P}9: \quad \max_{\theta} \; |a^H\theta|^2 \tag{9a}$$

$$\text{s.t.} \quad |\theta_{l,n}| = 1 \tag{9b}$$

$$arg(a^H\theta) = \varphi_0 \tag{9c}$$

It is not difficult to show that the optimal solution to P9 is $\theta^* = e^{j(\varphi_0 - arg(a^H))} = e^{j(\varphi_0 - arg(W^H G_b\bar{h}))}$. Thus $\phi^*_{l,n} = \varphi_0 - arg(W^H G_{b,l,n}) - arg(\bar{h}_{l,n})$ where $\bar{h}_{l,n}$ is the nth column vector of the Lth block of G_b.

Note that $W^H G_{b,l,n}$ combines the BS-IRS channel land the receive beamforming which can be regarded as the effective channel perceived by the nth reflecting element of the Lth IRS. It suggests that the nth phase shift of the Lth IRS should be tuned such that the phase of the signal that passes through the User-IRS and BS-IRS links is aligned with that of the signal over the user-BS direct link to achieve coherent signal combining at the BS. Thus, $P^* = \frac{\gamma\sigma^2}{|W^H(h_b+G_b\bar{h}\theta^*)|^2}$.

② Give θ, optimize W

$w^* = (h_b + G_b\bar{h}\theta)^H/||h_b + G_b\bar{h}\theta||$.

IV. Multiuser System

This section describes general multiuser system settings. Specifically, we propose an efficient algorithm to solve as a suboptimal solution by generalizing two approaches in a single-user case.

A. Alternating Optimization Algorithm

Give P_k

① For give W_k

$$\mathcal{P}10: \quad Find\ \theta \tag{10a}$$

$$\text{s.t.} \quad \frac{|W_k^H(h_{b,k}+G_b\bar{h_k}\theta)|^2 P_k}{\sum_{j\neq k}|W_k^H(h_{b,j}+G_b\bar{h_j}\theta)|^2 P_j + \sigma_k^2} \geq \gamma_k \tag{10b}$$

$$|\theta_{l,n}| = 1 \tag{10c}$$

The non-convex constraint (29) can be reformulate as below

$$\frac{P_k}{\gamma_k}[W_k^H(h_{b,k}+G_b\bar{h_k}\theta)(h_{b,k}+G_b\bar{h_k}\theta)^H W_k] - \sum_{j\neq k}[W_k^H(h_{b,j}+G_b\bar{h_j}\theta)(h_{b,j}+G_b\bar{h_j}\theta)^H W_k]P_j \geq \sigma_k^2$$

$$= \frac{P_k}{\gamma_k}A - \sum_{j\neq k}B$$

In the above expression, matrix A can be simply expressed as the product of the following matrices.

$$A = \theta^H\bar{h}_k^H G_b^H W_k W_k^H G_b\bar{h}_k\theta + \theta^H(\bar{h}_k^H G_b^H W_k W_k^H h_{b,k}) + (\bar{h}_k^H G_b^H W_k W_k^H h_{b,k})^H\theta + ||W_k^H h_{b,k}||^2$$

$$= \begin{bmatrix}\theta & t\end{bmatrix}\begin{bmatrix}\bar{h}_k^H G_b^H W_k W_k^H G_b\bar{h}_k & \bar{h}_k^H G_b^H W_k W_k^H h_{b,k} \\ \bar{h}_k^H G_b^H W_k W_k^H h_{b,k} & 0\end{bmatrix}\begin{bmatrix}\theta \\ t\end{bmatrix} + ||W_k^H h_{b,k}||^2$$

$$= \bar{\theta}^H R_{k,k}\bar{\theta} + ||W_k^H h_{b,k}||^2$$

B may be similarly represented in the same way as A. Then the problem P3 defined above is then converted to the problem below.

$$\mathcal{P}11: \quad Find\ \bar{\theta} \tag{11a}$$

$$\text{s.t.} \quad \frac{P_k}{\gamma_k}(\bar{\theta}^H R_{k,k}\bar{\theta} + ||W_k^H h_{b,k}||^2) - \sum_{j\neq k}(\bar{\theta}^H R_{k,j}\bar{\theta} + ||W_k^H h_{b,j}||^2)P_j \geq \sigma_k^2 \tag{11b}$$

$$|\theta_{l,n}|^2 = 1 \tag{11c}$$

Since $\bar{\theta}^H R_{k,j}\bar{\theta} = Tr(R_{k,j}\bar{\theta}\bar{\theta}^H) = Tr(R_{k,j}\Phi), \quad \Phi \succeq 0, rank(\Phi) = 1$

$$\mathcal{P}12: \quad Find\ \Phi \tag{12a}$$

$$\text{s.t.} \quad \frac{P_k}{\gamma_k}[Tr(R_{k,k}\Phi) + ||W_k^H h_{b,k}||^2] - \sum_{j\neq k}[Tr(R_{k,j}\Phi) + ||W_k^H h_{b,j}||^2)P_j \geq \sigma_k^2 \tag{12b}$$

$$\Phi_{t,t} = 1, \quad t = 1, \cdots, LN, LN+1 \tag{12c}$$

$$\Phi \succeq 0 \tag{12d}$$

The problem P12 described above can be solved through CVX because it is a convex problem. Find the feasible set of Φ satisfying constant (35) through CVX, and then find the values satisfying (36) and (37) through Gaussian randomization.

② For give θ

$$\mathcal{P}13: \quad \min_{W_k} \sum_{k=1}^{K} P_k \tag{13a}$$

$$\text{s.t.} \quad \frac{P_k}{\gamma_k}[W_k^H(h_{b,k} + G_b\bar{h_k}\theta)(h_{b,k} + G_b\bar{h_k}\theta)^H W_k]$$

$$- \sum_{j\neq k}[W_k^H(h_{b,j} + G_b\bar{h_j}\theta)(h_{b,j} + G_b\bar{h_j}\theta)^H W_k]P_j \geq \sigma_k^2 \tag{13b}$$

$$||w_k||^2 = 1 \tag{13c}$$

Since this problem P13 is the feasibility test, it can be converted to a problem that finds W_k that satisfies all those constraints. The constraint (39) can be reformulate as below: $Tr(W_k^H(h_{b,k} + G_b\bar{h_k}\theta)(h_{b,k} + G_b\bar{h_k}\theta)^H W_k) = Tr((h_{b,k} + G_b\bar{h_k}\theta)(h_{b,k} + G_b\bar{h_k}\theta)^H W_k W_k^H)$. And then define $V_k = W_k W_k^H$, that satisfing $rank(V_k) = 1, \quad Tr(V_k) = 1, \quad V_k \succeq 0$. $R_k = (h_{b,k} + G_b\bar{h_k}\theta)(h_{b,k} +$

$G_b\bar{h_k}\theta)^H, \quad R_j = (h_{b,j} + G_b\bar{h_j}\theta)(h_{b,j} + G_b\bar{h_j}\theta)^H$. Using the defined $V_k, \ R_k, \ R_j$ The previously defined issue P13 is redefined by the SDR as follows

$$\mathcal{P}14: \quad Find \ V_k \tag{14a}$$

$$\text{s.t.} \quad \frac{P_k}{\gamma_k}Tr[R_kV_k] - \sum_{j\neq k} P_jTr[R_jV_k] \geq \sigma_k^2 \tag{14b}$$

$$Tr(V_k) = 1 \tag{14c}$$

$$V_k \succeq 0 \tag{14d}$$

$$rand(V_k) = 1 \tag{14e}$$

The problem P14 described above also can be solved through CVX because it is a convex problem. Find the feasible set of Φ satisfying constant (42) through CVX, and then find the values satisfying (43),(44), and (45) through Gaussian randomization.

We propose an algorithm to find the optimal P while iteratively performing the described process in two previous steps.

Algorithm 1: Alternating Optimization Algorithm

1: Initialize the phase shifts W_k = W_k^1 and set the iteration number r = 1.

2: **Repeat**

3: Solve problem P12 for given W_k^r, and denote the solution after performing Gaussian randomization as θ^r.

4: Solve problem P14 for given θ^r, and denote the solution after performing Gaussian randomization as $W_k^r + 1$.

5: Update $r = r + 1$

6: **Until** The fractional decrease of the objective value is below a threshold > 0 or problem P12, P14 becomes infeasible.

V. Simulation Results

In this section, we want to validate the performance of the proposed scheme by presenting simulation results. In simulations, we compared the proposed SDR-based algorithm with the random phase shift decision algorithm. Simulations were carried out to measure total transmission power by changing the number K and SINR target gamma of the user device, respectively.

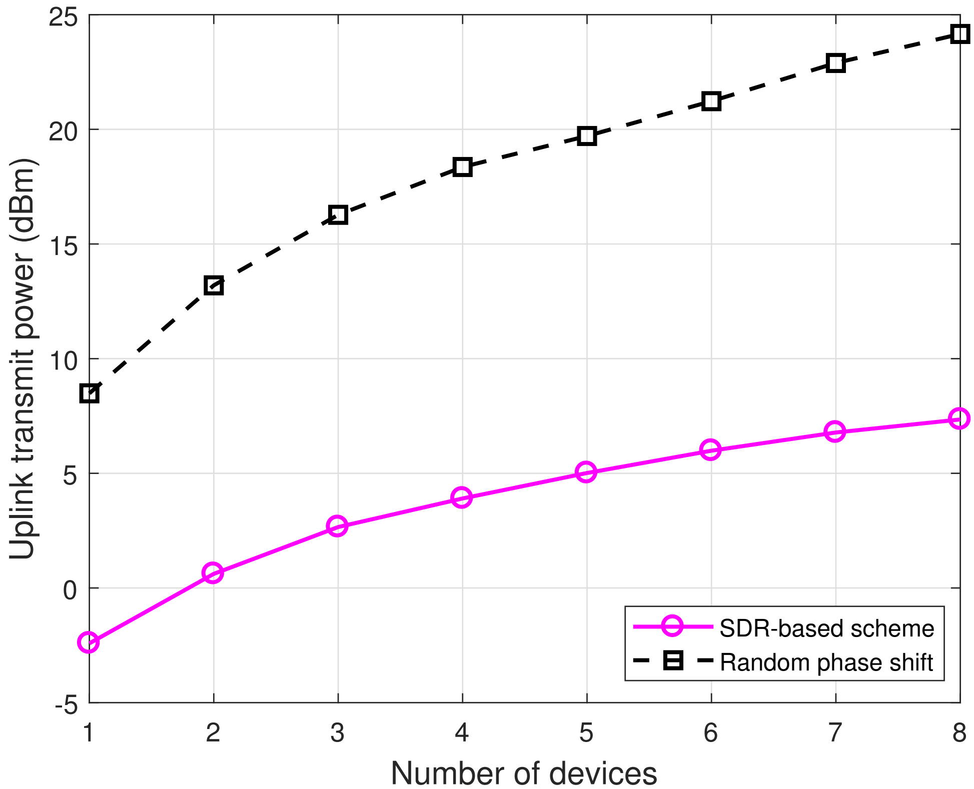

Fig. 1: Uplink Transmit Power Versus Number of devices

1) Uplink Transmit Power Versus Number of devices :
As shown in Fig.1, we compared the total uplink transmit power by changing the number of user devices and found that the total transmit power gradually increases as the number of devices increases. And crucially, we confirm that our proposed SDR-based algorithm consumes less power than the algorithm that randomly determines IRS phase shift.

2) Uplink Transmit Power Versus SINR target:
The results of the simulation, which was carried out by changing the target SINR of all users, were shown in Fig.2. As the target SINR increases, the total uplink transmission power of all users also increases, as does the transmission power of each user. And in this simulation, we also find that the SDR-based algorithm performs significantly higher than the random phase shift scheme.

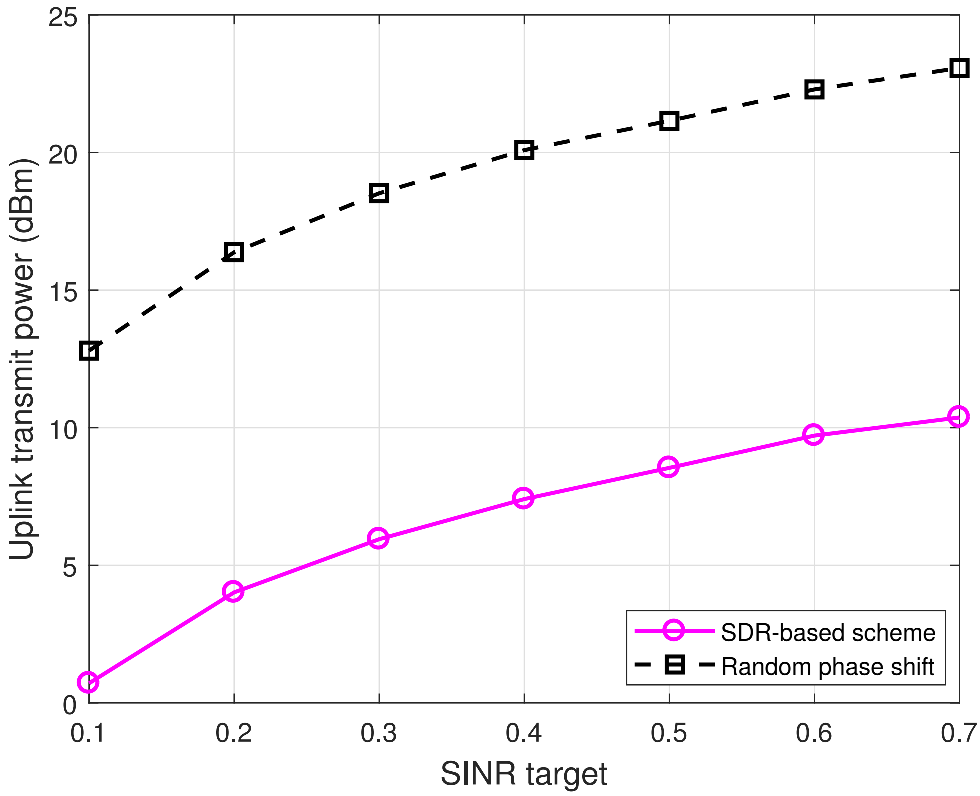

Fig. 2: Uplink Transmit Power Versus SINR target

VI. Conclusions

In this paper, we propose a novel approach that leverages manual IRS by smartly tuning signal reflection to increase frequency efficiency and energy efficiency and reduce the implementation cost of future wireless communication systems. Specifically, given the user's SINR constraints, active transmission beamforming of APs and manual reflection beamforming of IRSs are jointly optimized to minimize transmission power of IRS-enabled multi-user systems. An efficient algorithm is proposed to compromise between system performance and computational complexity by applying SDR and alternative optimization techniques. Simulations show better performance compared to schemes that randomly determine beamforming. The results of this work confirm that IRS not only minimizes transmission power, but also provides benefits of improving transmission data transfer rates and channel capacity.

VII. Reference

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